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Solutions to written exam for the M. Sc in Economics Economics of Exchange Rates

August 30, 2017

Number of questions: This exam consists of 2 questions.

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1. **Evaluation of fixed and flexible exchange–rate regimes and monetary unions**

This question relates to the following learning objectives: Knowledge: Describe the institutional features of the foreign exchange market products (spot and forward contracts) and be able to distinguish between speculation and arbitrage. Skills: Describe the main models of exchange rate determination (the Monetary approach to the exchange rate, Dornbusch overshooting model, the portfolio balance model and Lucas asset pricing model) and apply these models to analyze the effects of monetary and fiscal policy on the exchange rate, and summarize the empirical evidence on these models.

Consider the following open–economy model:

$$Md_t = Pi_t + \eta Y_t - \sigma r_t + U_t^1$$

$$Pi_t = \alpha P_t + (1-\alpha)(S_t + P_t^*)$$

$$Yd_t = \theta(s_t + P_t^* - P_t) - \beta\left(r_t + P_t - P_{t+1|t}\right) + \pi Yn + U_t^2$$

$$Ys_t = \phi(P_t - W_t) + U_t^3$$

$$O(P) = (P - Pn)^2$$

where notation is standard. Assume that $\eta(\theta+\beta) > \alpha$.

(a) Give a brief interpretation of the main assumptions and economic mechanisms underlying the equations.

Answer: The open economy AD-AS model statecd int he question is used to evaluate the choice between fixed and flexible exchange rates. The main underlying assumption is that the economy is affected by three different types of shocks, a money demand shock, an aggregate demand shock and an aggregate supply shock.

The first equation is a standard money demand function where the real balance is a function of output and the interest rate and U_t^1 is the money demand shock. Note that the price level in the demand function reflects both domestic prices and prices on imports as given by the third equation. Higher output levels and lower interest rates tend to increase the money demand, higher output increases the transaction demand whereas a lower interest rate reduces the return on bonds making it less attractive to hold bonds. The money demand shock is a white noise sequence implying that the shock has mean zero, a constant variance and is not autocorrelated.

The expression for the aggregate price level includes the parameter α which is the weight of the domestic price level in the aggregate price level and is a measure of openness, a small α implies a more open economy.

Aggregate domestic demand is given in the next equation. As can be seen in the equation, an appreciation tends to reduce aggregate demand (reduced foreign demand for domestic goods). A higher real interest rate also tends to reduce aggregate demand. A higher natural level of output implies a higher aggregate demand, higher potential output implies higher income. The supply shock is, as the demand shock, a white noise sequence.

The aggregate supply function is also standard, aggregate supply is inversely related to the real wage. A higher real wage reduces aggregate supply. The supply shock is a white noise sequence, i.e., has the same properties as the other two shocks.

We also assume that $\eta\,(\theta+\beta)>\alpha$. This assumption implies that the money demand curve is steeper than the aggregate demand curve. In other words, we assume a small open economy. The elasticity of domestic demand with respect to the real exchange rate and the real interest rate are relatively large compared to α.

In the objective function, the last equation, we assume that the authority only cares about the price level, the objective only involves domestic price stability. The authority wants to minimize deviations from the target price level.

In addition to the equations, we also assume that capital is perfectly mobile and abstract from risk aversion such that UIP holds (and the risk premium is zero). Labor contracts imply that the wage rate is set in the absence of shocks, they are set such that the expected output is equal to the natural level of output (the full employment output). Then the shocks affect the economy leading to short–run deviations of the price level from its target.

(b) Derive the aggregate demand curve, the money demand curve and the aggregate supply curve and illustrate the model in the price–output plane. Comment!

Answer: To derive the *aggregate demand* schedule we use the aggregate demand function above and solve for P_t.

$$P_t=\frac{\Theta}{\Theta+\beta}\left(s_t+P_{t^*}\right)-\frac{\beta}{\Theta+\beta}\left(r_t-P_{t+1|t}\right)+\frac{\pi}{\Theta+\beta}Yn-\frac{1}{\Theta+\beta}Yd_t-\frac{1}{\Theta+\beta}Ut_2$$

and then we find that the slope of this curve is $-\frac{1}{\Theta+\beta}$.

To derive the *money demand* schedule we use the money demand function and insert the expression for the price level Pi_t and then we solve for P_t.

$$P_t=-\frac{\eta}{\alpha}Y_t+\frac{\sigma}{\alpha}r_t+\frac{1}{\alpha}Md_t-(\frac{1}{\alpha}-1)(s_t+P_{t^*}-\frac{1}{\alpha}Ut_2)$$

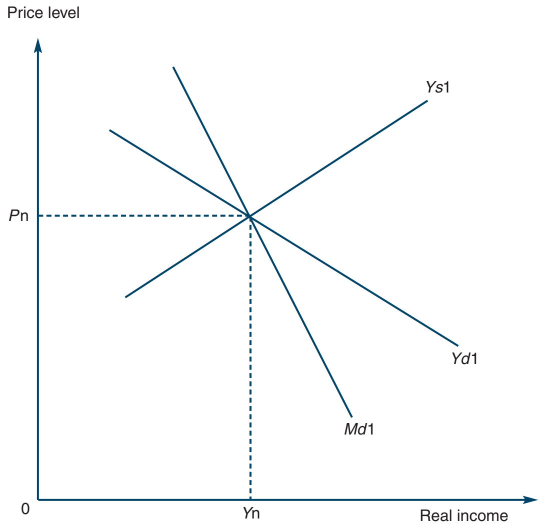
with slope equal to $-\frac{\eta}{\alpha}$.

The *aggregate supply* curve can be found using the aggregate supply function and solve for the price level P_t.

$$P_t = \frac{1}{\phi} Y s_t + W_t - U t_3$$

with slope $\frac{1}{\phi}$. Note that if $\phi \to 0$, then the slope approaches ∞ (a vertical line).

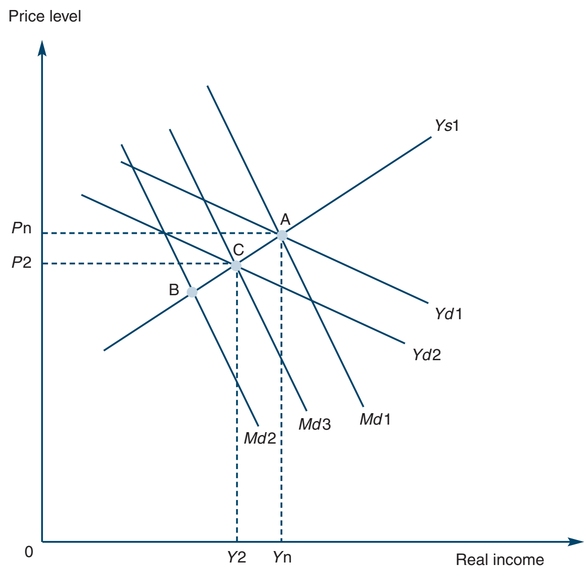
The model is then illustrated in the graph below.



(c) What is the optimal exchange rate regime if the economy is affected by aggregate demand or money demand shocks? Explain carefully!

Answer: Consider first the effects of a money demand shock. If the economy is hit by an unanticipated positive money demand shock there will be excess demand for money which causes a shift down of the Md–curve and the exchange rate will appreciate. What happens next depends on whether the exchange rate is fixed or flexible.

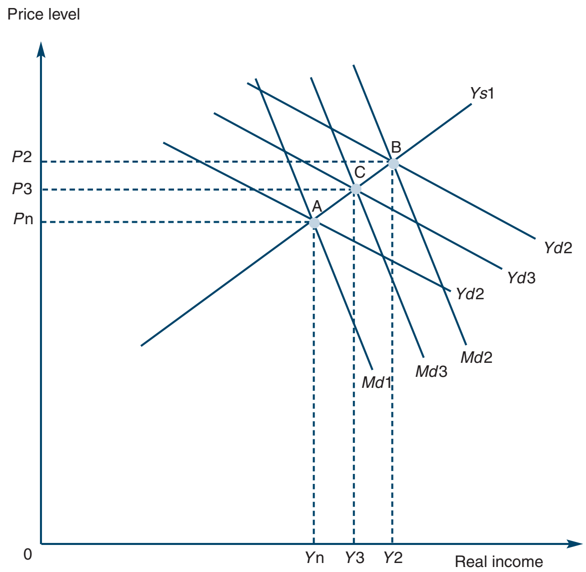
If the exchange rate is fixed, the excess demand for money causes a shift down of the Md–curve, a shift down from Md_1 to Md_2. The Central Bank (CB) must then intervene on the foreign exchange market in order to defend the fixed exchange rate. The CB buys foreign bonds and sells domestic money which expands the monetary base. As the monetary base expands, the money demand shifts back to its initial position, from Md_2 to Md_1. The total effect is then that neither prices nor output changes under fixed exchange rates, the economy is insulated to money demand shocks. This is illustrated in the graph below.



If the exchange rate is floating, the excess demand for money causes a shift down of the Md–curve, a shift down from Md_1 to Md_2. When the exchange rate appreciates, aggregate demand will shift down since less foreign demand leads to a fall in aggregate demand for domestic goods. The Yd-curve shifts from Yd_1 to Yd_2. From UIP we know that the interest rate must increase leading to lower money demand and the Md–curve shifts up from Md_2 to Md_3. The graph above illustrates. The new equilibrium is at point C. Here we note that the price level will deviate from the initial level and therefore it is optimal to have a fixed exchange rate in this case.

Next, we consider the effects of a positive aggregate demand shock. This leads to a shift up in the Yd–curve and as a result, there will be excess demand for money which will cause the exchange rate to appreciate.

In case of fixed exchange rates, the CB has to intervene by selling domestic money and buying foreign bonds. The Md–curve shifts up to Md_2 and there is a new short–run equilibrium at point B, see the graph below.



If the exchange rate is floating, the excess demand for money leads to an appreciation and therefore a shift down in the Yd–curve from Yd_2 to Yd_3 caused by a lower export demand. The Md–curve will shift up to Md_3 since the interest rate has increased (again through UIP) and this will reduce Md and we obtain a new short–run equilibrium at point C, see the graph above. Now we find that floating exchange rate is optimal if the economy is affected by aggregate demand shocks since the change in prices is smaller under floating than under fixed rates.

(d) Assume now that $\eta(\theta + \beta) < \alpha$. Illustrate the model graphically. What is the optimal exchange rate regime if the economy is affected by aggregate demand or money demand shocks? Explain carefully!

Answer: First, we need to remember that there will be excess demand for money above the Md–curve and that there will be excess demand for goods above the Yd–curve. Since $\eta(\theta + \beta) < \alpha$ we now have the case of a large closed economy and this implies that the Yd–curve is steeper than the Md–curve. This will not change our predictions above under the assumption of a small open economy. This can easily be seen if the money demand and the aggregate demand curves switch places.

If we assume that the economy is affected by the money demand shock, there will be excess demand for money as above and the Md-curve shifts down and the exchange rate will appreciate. The new Md-curve and the initial supply curve crosses at a point below the aggregate demand curve as above. What happens next depends on whether the exchange rate is fixed or flexible but the main conclusion from above will be unchanged.

The analysis of the effects of the shocks is identical to the case above for a small economy. The Md and Yd curves will move exactly as in the previous question and we find that fixed exchange rates are optimal when the economy is hit by a money demand shocks whereas floating exchange rates are optimal when aggregate demand shocks affect the economy. The reason for this is that movements the economy is in the same type of market condition in both cases, all points above the Md-curve implies excess demand for money and all points above the aggregate demand curve indicates excess demand for goods.

(e) Does the exchange rate regime affect the macroeconomy? Summarize the main lessons from the empirical literature.

Answer: The short answer is no. There is no consensus int he literature. Some researchers find that inflation is significantly lower under fisxed exchange rates than in countries having floating or intermediate floating exchange rates. This result is often motivated or explained by increased credibility of the central bank. Economic growth, on the other hand, seems to be higher in countries having floating exchange rates and that the volatility of growth tends to be higher under fixed exchange rates. These and other results are heavily dependent on the classification of exchange rate regimes, whether they use de facto or de jure regimes. The sample is also important, both the sample of countries and the time period examined. For example, the textbook presents a table showing that both inflation and economic growth tend to be lower under floating exchange rates but the differences are small.

2. **Exchange rate determination and microstructure**

This question relates to the following learning objectives: Describe the main models of exchange rate determination (the Monetary approach to the exchange rate, Dornbusch overshooting model, the portfolio balance model and Lucas asset pricing model) and apply these models to analyze the effects of monetary and fiscal policy on the exchange rate, and summarize the empirical evidence on these models, and Describe and apply microstructure based models to analyze price determination on the foreign exchange market and summarize the empirical evidence on these models.

Consider the the standard flexible price monetary model (FPMM)

$$s_t = (m_t - m_t^*) - \kappa (y_t - y_t^*) + \theta (i_t - i_t^*) \tag{1}$$

and the UIP relation

$$i_t - i_t^* = \mathbb{E}[\Delta s_{t+1} \mid \Omega_t] \tag{2}$$

where notation is standard.

(a) Explain the rationale behind the FPMM model.

Answer: The first equation states that the exchange rate is determined by relative money demands, relative output and the interest differential. To derive this equation we assume that PPP holds and that money demand is given by a standard money demand function where we also assume that all elasticities are equal across the two countries. By using the money demand functions (solved for the price level) and PPP we obtain the first equation. The second equation is the UIP relation where we have assumed that agents are risk neutral such that there is no risk premium.

(b) Show that the model above can be written as

$$s_t = (1-b)\sum_{i=0}^{\infty} b^i \mathbb{E}\left[f_{t+i} \mid \Omega_t\right] \tag{3}$$

where

$$f_{t+i} = \left(m_{t+i} - m^*_{t+i}\right) - \kappa\left(y_{t+i} - y^*_{t+i}\right)$$

and

$$b = \frac{\theta}{1+\theta}$$

Answer: Insert the UIP relation into the exchange rate equation and let $f_t = (m_t - m^*_t) - \kappa\,(y_t - y^*_t)$

$$s_t = f_t + \theta \mathbb{E}\left[\Delta s_{t+1} \mid \Omega_t\right]$$

which can be written as

$$s_t = \frac{1}{1+\theta} f_t + \frac{\theta}{1+\theta}\mathbb{E}\left[s_{t+1} \mid \Omega_t\right]$$

Let $b = \frac{\theta}{1+\theta}$ and solve this difference equation to obtain

$$s_t = (1-b)\sum_{i=0}^{\infty} b^i f_{t+i} + \lim_{j\to\infty} b^j s_{t+j}$$

where we note that the limit term goes to zero. The main argument is that we by imposing this assumption excludes bubbles.

(c) Explain how the spot exchange rate is determined according to the microstructure approach. What are the main assumptions about information available to market participants?

Answer: The foreign exchange market consists of two markets, the retail market where investors and dealers trade and the interbank market where dealers and the broker trade. Each market has its own trading mechanism. Customers place currency orders on the retail market and provide private information the dealers. Customer orders come from many different sources and may be generated by allocative, speculative and risk–management factors. Customer orders that is a function of current and past prices are termed feedback orders. Banks fill customer orders in the retail market. Dealers trade directly and indirectly in the interbank market and quote prices and initiate trades but no dealer has complete information about the state of the interbank market and they do not observe the structure of limit orders that describe the liquidity on the market. Brokers provide market–wide information on quotes and transaction prices.

Direct interdealer trading takes place simultaneously but dealers only have information about their own trades and they face constraints on both the duration and the size of their asset positions and overnight positions are typically zero or small.

Customer orders on the retail market provides important private information to dealers. Dealers working at banks with large customer base and a worldwide reporting system have informational advantages over other market participants (cf. Cheung and Chinn).

All dealers quote the same spot price to both other dealers and to their customers (quotes are publicly announced) based on common or public information. Dealer trade using the private information they receive from their own customers and are thus transmitting this information to other dealers. When public information has become public, it will be incorporated into prices. Microstructure models explain how private information becomes public information.

(d) Assume that the information set Ω_t above only includes public information. Show that the exchange rate equation in question (b) can be rewritten as

$$\Delta s_{t+1} = \frac{1-b}{b}\left(s_t - \mathbb{E}\left[f_t \mid \Omega_t^D\right]\right) + \varepsilon_{t+1}$$

where

$$\varepsilon_{t+1} = \frac{1-b}{b}\sum_{i=1}^{\infty} b^i \left(\mathbb{E}\left[f_{t+i} \mid \Omega_{t+1}^D\right] - \mathbb{E}\left[f_{t+i} \mid \Omega_t^D\right]\right).$$

Answer: We do this in two steps, first we derive the unexpected change at time $t+1$ ($s_{t+1} - \mathbb{E}\left[s_{t+1} \mid \Omega_t^D\right]$) in the exchange rate and then the expected change between t and $t+1$ ($\mathbb{E}\left[s_{t+1} \mid \Omega_t^D\right] - s_t$).

Start with the unexpected change $s_{t+1} - \mathbb{E}\left[s_{t+1} \mid \Omega_t D\right]$. We know that

$$s_{t+1} = (1-b)\sum_{i=0}^{\infty} b^i \mathbb{E}\left[f_{t+1+i} \mid \Omega_{t+1}^D\right]$$

where Ω_{t+1}^D is the information available to dealers when they set the price in period $t+1$. This expression can be rewritten as

$$s_{t+1} = \frac{1-b}{b}\sum_{i=1}^{\infty} b^i \mathbb{E}\left[f_{t+i} \mid \Omega_{t+1}^D\right]$$

Take the expected value of this expression given all information available at time t. We then obtain

$$\mathbb{E}\left[s_{t+1} \mid \Omega_t^D\right] = \frac{1-b}{b}\sum_{i=1}^{\infty} b^i \mathbb{E}\left[f_{t+i} \mid \Omega_t^D\right]$$

and then, finally, we take the difference between the actual value and the expected value

$$s_{t+1} - \mathbb{E}\left[s_{t+1} \mid \Omega_t^D\right] = \frac{1-b}{b}\sum_{i=1}^{\infty} b^i \left(\mathbb{E}\left[f_{t+i} \mid \Omega_{t+1}^D\right] - \mathbb{E}\left[f_{t+i} \mid \Omega_t^D\right]\right)$$

To derive the expected change we use the result above to find that

$$\mathbb{E}\left[s_{t+1} \mid \Omega_t^D\right] - s_t = \frac{1-b}{b}\sum_{i=1}^{\infty} b^i \mathbb{E}\left[f_{t+i} \mid \Omega_t^D\right] - (1-b)\sum_{i=0}^{\infty} b^i \mathbb{E}\left[f_{t+i} \mid \Omega_t^D\right]$$

$$= \frac{1-b}{b}\left(\sum_{i=1}\infty \sum_{i=1}^{\infty} b^i \mathbb{E}\left[f_{t+i} \mid \Omega_t^D\right] - b\sum_{i=0}^{\infty} b^i \mathbb{E}\left[f_{t+i} \mid \Omega_t^D\right]\right)$$

$$\frac{1-b}{b}\left(\sum_{i=1}\infty \sum_{i=1}^{\infty} b^i \mathbb{E}\left[f_{t+i} \mid \Omega_t^D\right] - b\mathbb{E}\left[f_t \mid \Omega_t D\right] - b\sum_{i=1}^{\infty} b^i \mathbb{E}\left[f_{t+i} \mid \Omega_t^D\right]\right)$$

Add and subtract $(1-b)\,\mathbb{E}\left[f_t \mid \Omega_t^D\right]$ on the right hand side and simplify such that

$$\mathbb{E}\left[s_{t+1} \mid \Omega_t^D\right] - s_t = \frac{1-b}{b}\left((1-b)\sum_{i=0}^{\infty} b^i \mathbb{E}\left[f_{t+i} \mid \Omega_t^D\right] - \mathbb{E}\left[f_t \mid \Omega_t^D\right]\right)$$

We can now finally establish the result in the question.

$$\Delta s_{t+1} = \frac{1-b}{b}\left(s_t - \mathbb{E}\left[f_t \mid \Omega_t^D\right]\right) + \varepsilon_{t+1}$$

and

$$\varepsilon_{t+1} = \frac{1-b}{b}\sum_{i=1}^{\infty} b^i \left(\mathbb{E}\left[f_{t+i} \mid \Omega_{t+1}^D\right] - \mathbb{E}\left[f_{t+i} \mid \Omega_t^D\right]\right)$$

(e) Interpret the two equations in (d). Explain how these expressions relate to the microstructure approach.

Answer: The model above implies that we have decomposed a change in the spot exchange rate (or the quote) into two parts, the expected change and the unexpected change. According to the model, the spot exchange rate is determined by current and future fundamentals and include forecasts of future fundamentals given common knowledge Ω_t^D. Order flows contain information about future fundamentals that is not public, i.e., not in Ω_t^D. Order flows should predict future fundamentals beyond information contained in Ω_t^D. Dealers period–t quote must be based on public information known at time t, i.e., $\mathrm{E}\left[\Delta s_{t+1} \mid \Omega_t^D\right]$. Unexpected changes reflect new information arriving between the start of period t and $t+1$. But new information is only important if it revises dealers forecast of the present value of fundamentals based on common information, i.e., $\mathrm{E}\left[f_{t+i} \mid \Omega_{t+1}^D\right] - \mathrm{E}\left[f_{t+i} \mid \Omega_t^D\right]$.

Common knowledge will immediately be incorporated into quoted prices and from Cheung and Chinn we know that it usually takes less than a minute before new information is reflected in the price. Dealers interpret the common knowledge differently. Two dealers may not use the same model linking fundamentals to spot rates. New information which is common knowledge can be the source of dispersed information, i.e., be a source of customer order flows. Common knowledge may operate both via the direct channel (common knowledge to all dealers) and via the indirect channel (dispersed information) through order flows. Order flows have no effect on quotes because it is private information. But the private information is transmitted through interdealer orders. Dealers use private information to trade in the interbank market creating order flows between dealers which will transmit and make private information public (information aggregation).

(f) Summarize the empirical evidence on the relationship between order flows and spot exchange rates.

Answer: There is a strong positive contemporaneous correlation between daily changes in the price of FX and interdealer order flow. The correlation is robust to different forms of interdealer trading and appears across a wide cross-section of currencies. The contribution of interdealer order flows to daily changes in exchange rates is much higher than that found for any other macroeconomic or financial variables. The impact of order flows on exchange rates may depend on trading volume. The contemporaneous relationship between spot rate changes and order flow applies to both interdealer flows and customer flows. Customer flows disaggregated by customer type have more explanatory power for exchange rate returns than the aggregate flows received by individual banks. Disaggregated flows can account for less of the variation in exchange rate returns than aggregate interdealer order flows, but the explanatory power of customer and dealers flows are comparable at lower frequencies.